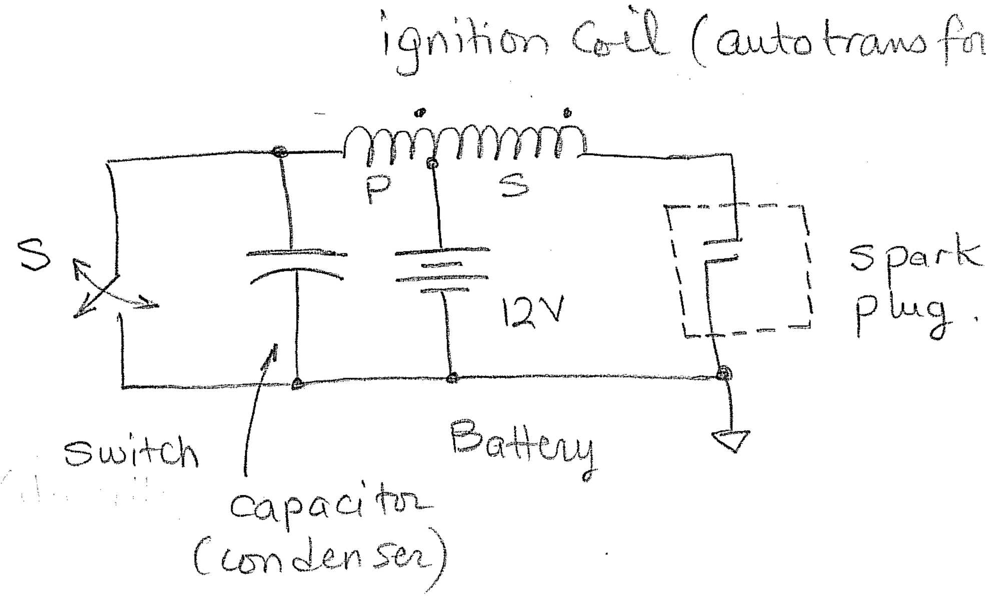
Chapter 8

Natural and Step Responses of RLC Circuits.

In this chapter we discuss the natural and step responses of circuits containing both inductors and capacitors. The discussion is limited to either series RLC or parallel RLC circuits.

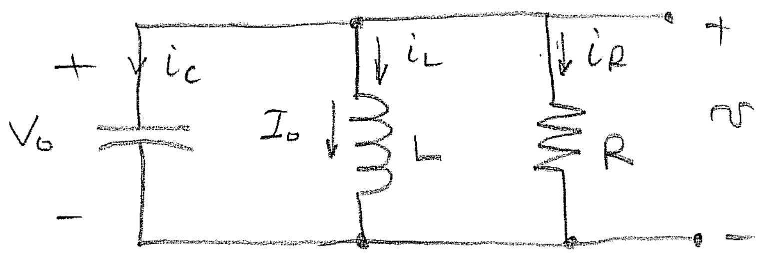
An automobile ignition circuit is based on the transient response of an RLC circuit as shown below:



The switch S causes a rapidly changing current in the primary winding (P) which induces a very high voltage in the secondary winding (S) up to 40 kV, which will ignite a spark across the gap of the spark plug. In older cars the switch was a relay, whereas in today's cars the switch is electronic rather than a mechanical relay.

8.1 Natural Response of a Parallel RLC Circuit

To find the natural response of a parallel circuit, shown in Fig. 8.1 below, we need to derive the differential equation that governs the voltage v. This obtained using KCL at the top node of the circuit:



(Fig. 8.1)

$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,d\tau + I_0 + C\frac{dv}{dt} = 0 \qquad (8.1)$$

$$(\; i_R \quad + \quad i_L \quad + \quad i_C \; = 0)$$

Note that I_0 is the initial current in L (i.e. $i_L(0)$) and that V_0 is the initial voltage across C (i.e. $v(0)$).

By differentiating Equation 8.1, dividing by C and rearranging we obtain:

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0. \qquad (8.3)$$

General Solution of the Second-Order Differential Equation

We note that the second derivative of the solution v, plus a constant times its first derivative, plus a constant times the solution itself, must be equal to zero for all values of t. The exponential function can satisfy this condition, and thus it is reasonable to assume that $v = Ae^{st}$ is a particular solution to 8.3 and that it must satisfy it for all values of t; so by substituting into (8.3) we get:

$$As^2e^{st} + \frac{As}{RC}e^{st} + \frac{Ae^{st}}{LC} = 0 \Rightarrow$$

$$Ae^{st}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right) = 0 \qquad (8.5)$$

So for Ae^{st} to be a solution, then the parenthesis term must be equal to zero!

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0 \qquad (8.6)$$

The above equation is called the <u>characteristic equation</u>. Its roots are given by:

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

If either root is substituted in the assumed particular solution we obtain a solution for 8.3 regardless of the value of A. Therefore, both

$$v_1 = A_1 e^{s_1 t} \quad \text{and} \quad v_2 = A_2 e^{s_2 t}$$

satisfy Eq. 8.3. Their sum is the solution of Eq. 8.3 as can be easily verified. So

$$v = v_1 + v_2 = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad (8.13)$$

The constants A_1 and A_2 are obtained from initial conditions on v and dv/dt at $t=0$.

The roots of the characteristic equations are usually written in the following form:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

where $\alpha = \frac{1}{2RC}$ is the <u>Neper frequency</u>

$\omega_0 = \frac{1}{\sqrt{LC}}$ is the <u>resonant frequency in radians</u>.

The values s_1 and s_2 are known as the <u>characteristic roots</u> or complex frequencies.

Three possible outcomes are possible:

i) Overdamped response: when $\alpha^2 > \omega_0^2$ both roots s_1 and s_2 are real and distinct.

ii) Underdamped response: when $\alpha^2 < \omega_0^2$ then s_1 and s_2 are complex conjugate values.

iii) Critically damped response: when $\omega_0^2 = \alpha^2$ then the roots s_1 and s_2 are real and equal.

(See Example 8.1)

Assessment Problem 8.1

The resistance and inductance of a parallel RLC circuit $R = 100\,\Omega$ and $L = 20\,mH$.

a) Find C that makes the response critically damped.

For critical damping $\alpha^2 = \omega_0^2 \Rightarrow \left(\frac{1}{2RC}\right)^2 = \frac{1}{LC}$

or $C = \frac{L}{4R^2} = \frac{20\times10^{-3}}{4\times 100^2} = 5\times10^{-7}\ F = 500\,nF$
$= 0.5\,\mu F$

b) If $\alpha = 5\times10^3$ rad/s (neper frequency), find C and the characteristic roots:

$$\alpha = \frac{1}{2RC} = 5\times10^3 \Rightarrow C = \frac{1}{2R\alpha} = \frac{1}{2\times100\times5\times10^3}$$
$$= 1\times10^{-6} = 1\mu F$$

To determine the nature of the response, we need to calculate $\frac{1}{LC}$ $(=\omega_0^2)$ and compare it to α^2:

$$\omega_0^2 = \frac{1}{20\times10^{-3}\times10^{-6}} = 50\times10^6$$

and $\alpha^2 = 25\times10^6$

Since $\alpha^2 < \omega_0^2$ then s_1 and s_2 are complex conjugate values:

$$s_1 = -5\times10^3 + j\sqrt{50-25}\times10^3$$
$$= -5000 + j\,5000 \text{ rad/s}$$

$$s_2 = -5000 - j\,5000 \text{ rad/s.}$$

c) If C is adjusted to give a resonant frequency of 20 krad/s, find C and the characteristic roots:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 20\times10^3 \Rightarrow \frac{1}{LC} = \omega_0^2 = 400\times10^6$$

$$C = \frac{1}{L\omega_0^2} = \frac{1}{20\times10^{-3}(20\times10^3)^2} = 0.125\times10^{-6}$$
$$= 0.125\,\mu F = 125\,nF$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times100\times0.125\times10^{-6}} = 4\times10^4 \text{ rad/s}$$

$\alpha^2 = 16\times10^8$

since $\alpha^2 (=16\times10^8) > \omega_0^2 (=4\times10^8)$ then

Then the response is overdamped with s_1 and s_2 being real distinct values:

$$S_1 = -4\times10^4 + \sqrt{16-4}\times10^4 = -5359 \text{ rad/s}$$

$$S_2 = -4\times10^4 - \sqrt{16-4}\times10^4 = -74\,641 \text{ rad/s}$$

8.2 The Forms of the Natural Response of a Parallel RLC Circuit

i) The overdamped response:

When $\alpha^2 > \omega_0^2$ the solution of the voltage is

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

with s_1 and s_2 being real and distinct. The constants A_1 and A_2 are found from the initial capacitor voltage and initial inductor current!

$$v(0^+) = A_1 + A_2 \qquad (8.19)$$

and

$$\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2 \qquad (8.20)$$

but

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \left(-\frac{V_0}{R} - I_0\right)\frac{1}{C}$$

The process of finding A_1 and A_2 is summarized:

1) Find the roots s_1 and s_2 from the values of R, L, and C.

2) Find $v(0^+)$ and $\frac{dv(0^+)}{dt}$ using circuit analysis

3) Find A_1 and A_2 using equations 8.19 and 8.20.

Study Examples 8.2 and 8.3!

Assessment Problem 8.2

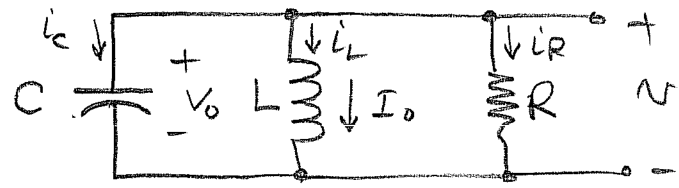
Use the integral relationship between i_L and v to find an expression for i_L in the following circuit:

$C = 0.2\,\mu F$

$L = 50\,mH$

$R = 200\,\Omega$

$V_0 = 12\,V$ and $I_0 = 30\,mA$



1) Calculate roots of the characteristic equation:

$$\alpha^2 = \left(\frac{1}{2RC}\right)^2 = \left(\frac{10^6}{2\times 200\times 0.2}\right)^2 = 156.25\times 10^6$$

$$\omega_0^2 = \frac{1}{LC} = \frac{10^6}{50\times 10^{-3}\times 0.2} = 100\times 10^6$$

Since $\alpha^2 > \omega_0^2$ the response is overdamped:

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -12.5\times 10^3 + \sqrt{156.25 - 100}\times 10^3$$

$$= -12500 + 7500 = -5000 \text{ rad/s}$$

$$S_2 = -12500 - 7500 = -20\,000 \text{ rad/s}$$

2) $v(0^+) = 12V$

$$\frac{dv(0^+)}{dt} = -\left(\frac{V_0}{R} + I_0\right)\frac{1}{C} = -90\times10^{-3} \times \frac{1}{0.2\times10^{-6}}$$

$$= -450\times10^{3} \text{ V/s.}$$

So $\quad A_1 + A_2 = 12$

$$-5000A_1 - 20000A_2 = -450\times10^{3}$$

$$A_1 = -14 \quad \text{and } A_2 = 26$$

Hence $\quad v(t) = -14e^{-5000t} + 26e^{-20000t}$ V.

$$i_L(t) = \frac{1}{L}\int_0^t v(t)\,dt + I_0$$

$$= \frac{1}{50\times10^{-3}}\int_0^t \left(-14e^{-5000t} + 26e^{-20000t}\right)dt + 30\times10$$

$$= \frac{1}{50\times10^{-3}}\left(\frac{14}{5000}e^{-5000t} - \frac{26}{20000}e^{-2000t}\right)\Big|_0^t + 30\times10$$

$$= 0.056e^{-5000t} - 0.026e^{-2000t} \text{ A} \quad t \geq 0.$$

$$= 56e^{-5000t} - 26e^{-20000t} \text{ mA} \quad t \geq 0$$

The Underdamped Voltage Response

When $\omega_0^2 > \alpha^2$ the roots of the characteristic equation are complex and the response is underdamped:

$$s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\omega_d$$

$$s_2 = -\alpha - j\omega_d$$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ is the damped frequency.

If we replace for s_1 and s_2 their values given above in equation 8.18 we obtain:

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$= A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha} e^{-j\omega_d t}$$

And by using Euler's identity ($e^{j\theta} = \cos\theta + j\sin\theta$) we get:

$$v(t) = e^{-\alpha t}\left[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t\right] \Rightarrow$$

$$v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t) \qquad (8.28)$$

Note that the constants B_1 and B_2 are real because the voltage is a real function. Note that A_1 and A_2 are complex conjugates.

For the underdamped case, the two equations from which we determine B_1 and B_2:

$$v(0^+) = V_0 = B_1 \quad \text{and} \quad \frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = -\alpha B_1 + \omega_d B_2$$

Note that the trigonometric functions indicate that the response is oscillatory at a frequency ω_d. Also the amplitude of the oscillation decreases exponentially. The constant α is also referred as damping factor or damping coefficient.

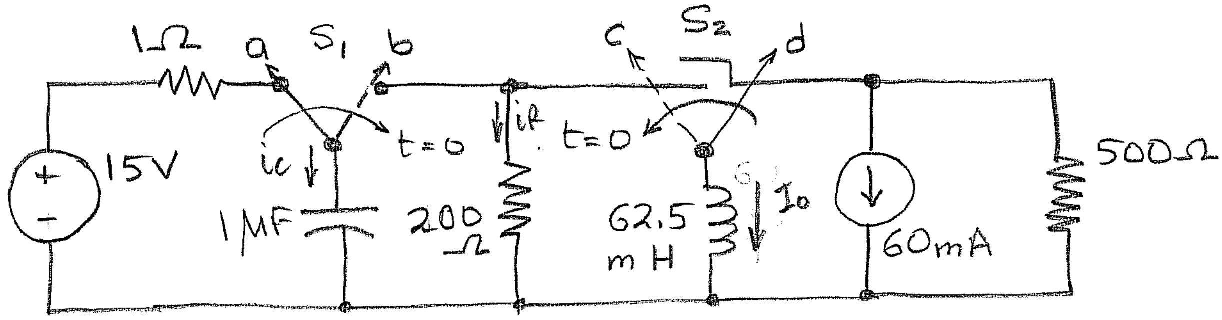
Note that when there is no resistance ($R \to \infty$) there is no damping and $\alpha = 0$.

The oscillatory response occurs because of the two types of energy storage elements, i.e. the inductor and capacitor.

Study Example 8.4 and solve Assessment problem 8.4. In what follows the solution of Problem 8.19 is given:

Problem 8.19

Consider the following circuit,



Switches S_1 and S_2 have been in positions a and d for a long time. At t=0 both switches move to positions b and c, respectively. Find $v(t)$ for $t \geq 0$.

Solution:

First find the neper and resonant frequencies:

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 200\times 10^{-6}} = 2.5\times 10^{+3} \text{ rad/s}$$

$$\alpha^2 = 6.25\times 10^6$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{62.5\times 10^{-3}\times 10^{-6}} = 16\times 10^6$$

Since $\alpha^2 < \omega_0^2$ the response is underdamped.

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 3.1225\times 10^3 \text{ rad/s.}$$

The response is given by:

$$v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

$B_1 = V_0 = 15\text{V}$ and $I_0 = -60\text{mA} = -0.06\text{A}$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = -\alpha B_1 + B_2\omega_d$$

$$i_c(0^+) = -\left(\frac{V_0}{R} + I_0\right) = -\left(\frac{15}{200} - 0.06\right) = -0.015$$

$$-2.5\times 10^3\times 15 + B_2\, 3122.5 = -\frac{0.015}{10^6} \Rightarrow$$

$$B_2 = 7.206$$

$$v(t) = e^{-2500t}(15\cos 3122.5t + 7.206\sin 3122.5t)$$

The Critically Damped Response

When $\alpha^2 = \omega_0^2$, the response is critically damped, i.e. it is on the verge of oscillation. The roots of the characteristic equation are real and equal:

$$s_1 = s_2 = -\alpha = -\frac{1}{2RC}$$

The solution of the DE takes the form

$$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t} \qquad (8.34)$$

The constants may be found from:

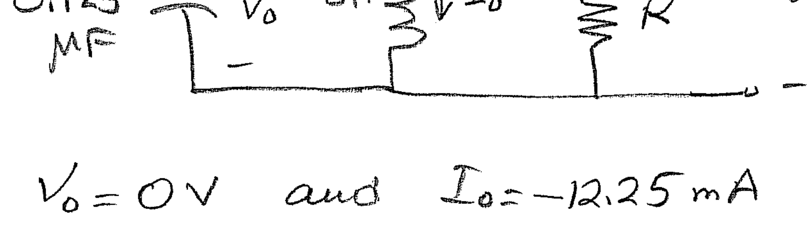
$$v(0^+) = V_0 = D_2 \qquad (8.35)$$

and

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2 \qquad (8.36)$$

Example 8.5

Consider the circuit of Example 8.4.



$V_0 = 0\text{ V}$ and $I_0 = -12.25\text{ mA}$

a) Find the value of R that results in critical damping:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \left(\frac{1}{8 \times 0.125 \times 10^{-6}}\right)^{\frac{1}{2}} = 1000 \text{ rad/s}$$

For critical damping $\alpha = \omega_0$:

$$\alpha = \frac{1}{2RC} = 1000 \Rightarrow R = \frac{1}{2C \times 1000} = 4000\,\Omega$$

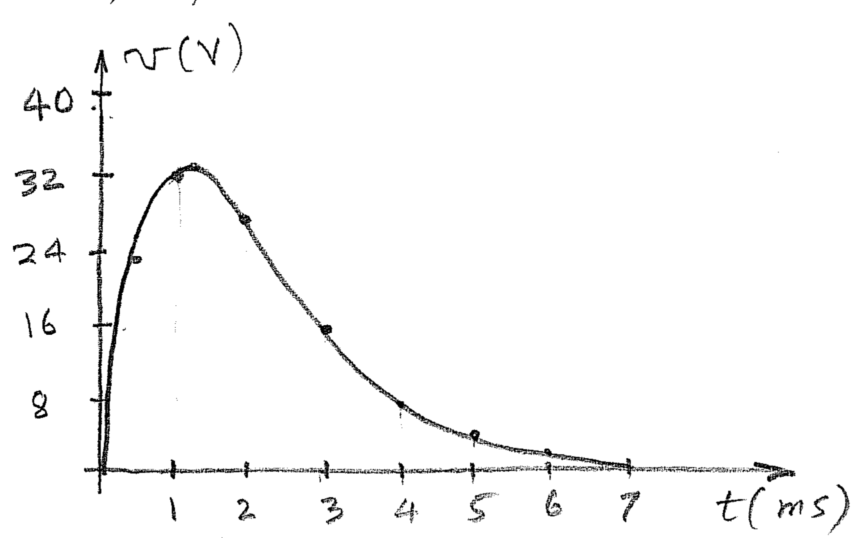
b) Calculate $v(t)$ for $t \geq 0$

$$D_2 = V_0 = 0$$

$$D_1 = \frac{i_c(0^+)}{C} = -\frac{I_0}{C} = \frac{12.25 \times 10^{-3}}{0.125 \times 10^{-6}} = 98000 \text{ V/s}$$

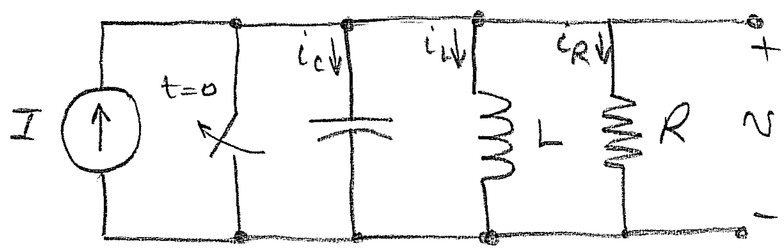
So $\quad v(t) = 98000\, t e^{-1000t} \quad V$

c) Plot $v(t)$ for $0 \leq t \leq 7$ ms



8.3 The Step Response of a Parallel RLC Circuit.

In this case the voltage across the parallel branches when a dc current source is applied as shown in the circuit below.



There may or may not be energy initially stored in L or C. By KCL we have:

$$i_L + i_R + i_C = I \qquad (8.37)$$

By expressing the currents in terms of v:

$$\frac{1}{L}\int_0^t v\,d\tau + \frac{v}{R} + C\frac{dv}{dt} = I$$

By differentiating and dividing by C we get:

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0 \qquad (8.43)$$

This is the same equation as that of the natural response. Thus the three possible solutions are:

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad (8.44)$$

$$v(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t \qquad (8.45)$$

$$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t} \qquad (8.46)$$

The current in the inductor is of particular interest because it does not approach zero at $t \rightarrow \infty$. Rather the current in L approaches I. To find i_L we use v as given by equations 8.44 to 8.46 and express i_R and i_C in 8.37 in terms of v. It can be easily verified that the three solutions for i_L are:

$$i_L = I + A_1' e^{s_1 t} + A_2' e^{s_2 t} \qquad (8.47)$$

$$i_L = I + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t \qquad (8.48)$$

$$i_L = I + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t} \qquad (8.49)$$

The constants in the above 3 equations can be found from $i_L(0)$ and $\frac{di_L(0)}{dt}$.

The second order DE of i_L can be obtained by noting that:

$$v = L \frac{di_L}{dt} \quad \text{and} \quad \frac{dv}{dt} = L \frac{d^2 i_L}{dt^2}$$

By replacing the above in equation (8.37) and dividing by LC we get:

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I}{LC} \qquad (8.50)$$

By comparing 8.50 with 8.43 we note the presence of a non-zero term on the rhs of the equation. Note that equations 8.47 to 8.46 are the solutions to 8.50.

In general the solution of a second-order DE with a constant forcing function equals the forced function plus a response function identical to the natural response. Thus

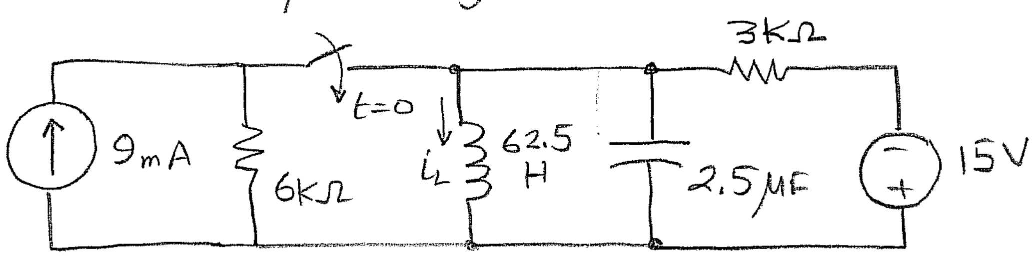
$i = I_f +$ natural response of i

$v = V_f +$ natural response of v

Study Examples 8.6 → 8.10.

Problem 8.32

Consider the following circuit:



The switch has been in the open position for a long time. It's closed at $t=0$. Find $i_L(t)$ for $t \geq 0$.

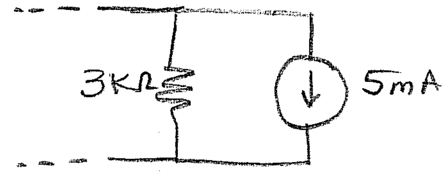
Solution:

Let us first the initial conditions for L and C.

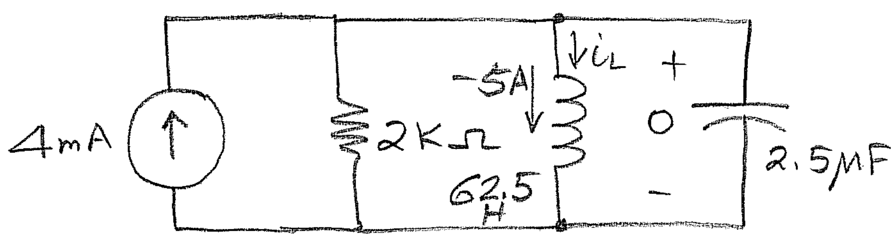
$V_o = 0\,V \qquad I_o = -\frac{15}{3} = -5\,mA$

The 3KΩ resistor and 15V source will be represented by a Norton equivalent:

The 3kΩ resistor will appear in parallel with the 6kΩ resistor, and the current source will



combine with the 9mA current source to give a net current source of $9-5=4$ mA. The net circuit is:



Let us find the characteristic roots:

$$\alpha = \frac{1}{2RC} = 100 \text{ rad/s} \Rightarrow \alpha^2 = 10^4.$$

$$w_0^2 = \frac{1}{LC} = 0.64 \times 10^4$$

So $\alpha^2 < w_0$ and the natural response is under-damped:

$S_1 = -100 + j\,60$ and $S_2 = -100 - j\,60$ rad/s.

So $$i_L(t) = I_f + B_1' e^{-\alpha t} \cos w_d t + B_2' e^{-\alpha t} \sin w_d t$$

$I_f = 4$ mA by circuit analysis

$$i_L(0^+) = I_f + B_1' = -5 \Rightarrow B_1' = -5-4 = -9 \text{ mA}$$

$$\frac{di_L(0^+)}{dt} = \frac{V_0}{L} = 0.$$

$$\frac{di_L}{dt} = B_1'(-\alpha)e^{-\alpha t}\cos\omega_d - B_1' e^{-\alpha t}\omega_d \sin\omega_d t$$
$$+ B_2'(-\alpha)e^{-\alpha t}\sin\omega_d t + B_2' e^{-\alpha t}\omega_d \cos\omega_d t$$

$$\frac{di_L(0)}{dt} = -\alpha B_1' + \omega_d B_2' = 0 \Rightarrow B_2' = \frac{\alpha}{\omega_d} B_1' \Rightarrow$$

$$B_2' = \frac{100}{60}(-9) = -15\,mA.$$

Hence

$$i_L(t) = 4 - 9e^{-100t}\cos 60t - 15e^{-100t}\sin 60t$$

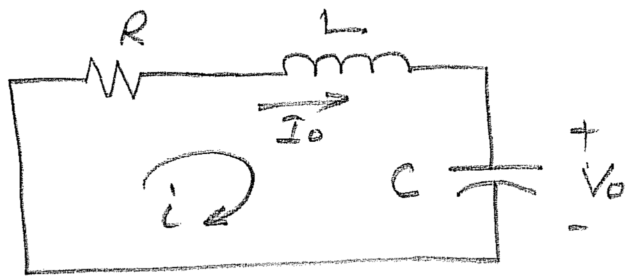
for $t \geq 0$.

One can verify that initial and final conditions match.

Solve Problems 8.33 to 8.36!

8.4 The Natural and Step Response of a Series RLC Circuit

The natural response of a series RLC circuit is in terms of the current i, as shown in the circuit below:



The mesh current equation (KVL) is:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau + V_0 = 0$$

Differentiate, divide by L and rearrange:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \qquad (8.54)$$

The characteristic equation of the above DE is:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

with roots

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

with $\alpha = \dfrac{R}{2L}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}}$

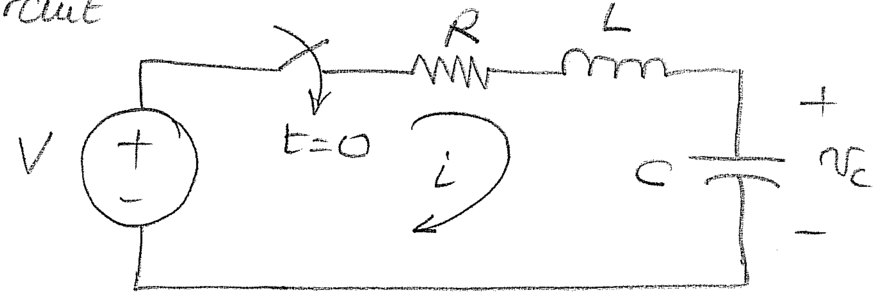
The response here will be overdamped, underdamped and critically damped:

So $i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (overdamped)

$i(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$ (underdamped)

$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ (critically damped).

The step response is described in terms of the capacitor voltage as illustrated using the following circuit



KVL gives:

$$Ri + L\frac{di}{dt} + v_c = V \qquad (8.63)$$

But $i = C\frac{dv_c}{dt}$ and $\frac{di}{dt} = C\frac{d^2 v_c}{dt^2}$

Substitute the above in equation (8.63), divide by L and rearrange:

$$\frac{d^2 v_c}{dt} + \frac{R}{L}\frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{V}{LC} \qquad (8.66)$$

The above equation has the same from as 8.50 and its three possible solutions are:

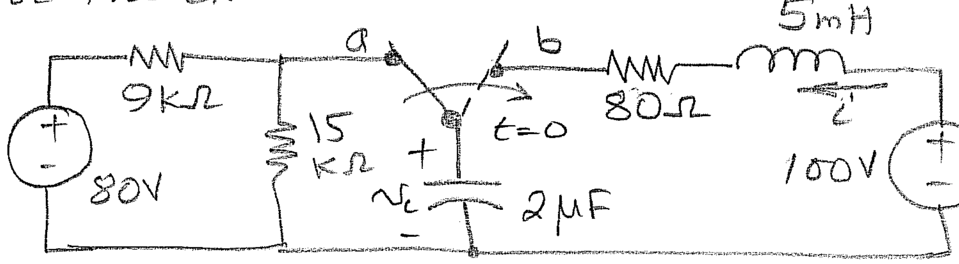
$v_c(t) = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$ (overdamped).

$v_c(t) = V_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$ (underdamped)

$v_c(t) = V_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$ (critically damped).

Study examples 8.11 and 8.12
Assessment Problems 8.7 and 8.8

For the circuit shown below:



The switch has been in position a for a long time; at t=0 the switch moves to position b.

Find a) $i(0^+)$, b) $v_c(0^+)$ c) $di(0^+)/dt$
d) S_1 and S_2 and e) $i(t)$ $t \geq 0$.

Solution:

a) $i(0^+) = 0A$

b) $v_c(0^+) = 80 \times \frac{15}{24} = 50V$

c) The initial current is 0 and so $v_R(0^+)$ is also zero. By KVL at 0^+:

$$v_c(0^+) + v_L(0^+) + v_R(0^+) = 100 \Rightarrow$$

$$v_L(0^+) = 100 - 50 = 50V$$

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{50}{5\times10^{-3}} = 10\,000 \text{ A/s}$$

d) $\alpha = \frac{R}{2L} = \frac{80}{10\times10^{-3}} = 8000$ rad/s. $\Rightarrow \alpha^2 = 64\times10^6$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{5\times10^{-3}\times 2\times10^{-6}} = 100\times10^6$$

$\alpha^2 < \omega_0^2$ so the response is underdamped and s_1 and s_2 are:

$$s_1 = -8000 + j\sqrt{100-64}\times10^3$$
$$= -8000 + j\,6000 \text{ rad/s}$$
$$s_2 = -8000 - j\,6000 \text{ rad/s}.$$

e) $i = I_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$

$I_f = 0$ by circuit analysis

$i(0^+) = 0 = B_1'$

$$\frac{di(0^+)}{dt} = B_2'\,\omega_d = 10\,000 \Rightarrow B_2' = \frac{10\,000}{6000} = 1.67$$

so $i(t) = 1.67\,e^{-8000t}\sin 6000t$ A for $t \geq 0$

AP 8.8: Find $v_c(t)$ for $t \geq 0$

$$v_c(t) = V_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t.$$

$V_f = 100\text{V}$ $\quad v_c(0^+) = 50\text{V}$ $\quad \frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = 0$

$100 + B_1' = 50 \Rightarrow B_1' = -50\text{V}.$

$$\frac{dv_c(0^+)}{dt} = -\alpha B_1' + B_2'\omega_d = 0 \Rightarrow B_2' = \frac{\alpha B_1'}{\omega_d} \Rightarrow$$

$$B_2' = -\frac{8000\times 50}{6000} = 66.7$$

$$v_c(t) = 100 - e^{-8000t}(50\cos 6000t + 66.7\sin 6000t)\text{V}$$